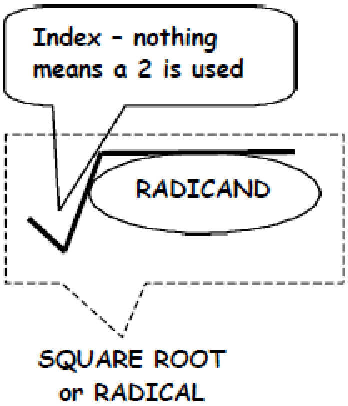
5.3 – Square Roots and Their Properties

Curriculum Outcomes	Related Activities	Page in Text
• use inductive and deductive reasoning when observing patterns, developing properties, and making conjectures	• use nested triangles to see that the effect of a dilatation from a vertex of a triangle is to create similar triangles • measure side lengths in similar triangles to observe constant ratios and generalize from the patterns	213 213
• determine whether differenced in measurement, while conducting experiments are significant or crucial	• compare ratios based on measurements which might be close to equal, ut not exactly equal, to decide if equality can be assumed	214
• determine and apply formulas for perimeter, area, surface area, and volume	• examine the effect of scale factors on triangle perimeter and area	215
• solve problems involving similar triangles and right triangles	• find missing lengths in triangles by setting up proportions with similar triangles	215
• solve problems involving measurement using bearings and vectors • apply the properties of similar triangles	• explore the equivalence of vector, compass direction, and bearing descriptions of movement	216

Number	Square
1	$1^2 = 1$
2	$2^2 = 4$
3	$3^2 = 9$
4	$4^2 = 16$
5	$5^2 = 25$
6	$6^2 = 36$
7	$7^2 = 49$
8	$8^2 = 64$
9	$9^2 = 81$
10	$10^2 = 100$



Perfect Square	Square Root
1	$\sqrt{1} = 1$
4	$\sqrt{4} = 2$
9	$\sqrt{9} = 3$
16	$\sqrt{16} = 4$
25	$\sqrt{25} = 5$
36	$\sqrt{36} = 6$
49	$\sqrt{49} = 7$
64	$\sqrt{64} = 8$
81	$\sqrt{81} = 9$
100	$\sqrt{100} = 10$

A perfect square is a number that when you find the square root, you do not get a decimal answer (you get a whole number for an answer). Examples are #1,4,9,16,25)

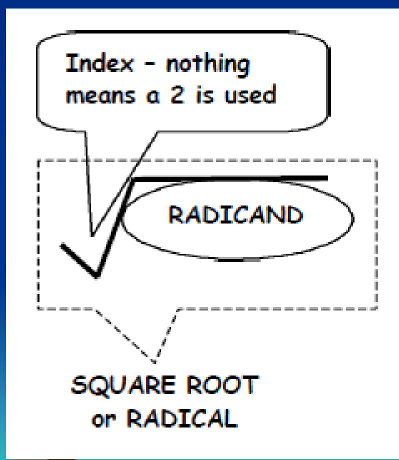
- When the Pythagorean theorem is applied to solve real-life problems, very often the side lengths of the right triangle are not whole numbers.
- For example: $6^2 + 4^2 = 36 + 16 = 52$
- Since 52 does not have a whole number square root, the length of the hypotenuse is not a whole number.

Square Root Notation

- The square root of 16 is 4 or -4.
- Notice that any whole number has two square roots:
- One positive
- One negative
- The positive root is called <u>the principle square root</u>.

Square Root Notation

- The symbol $\sqrt{n}$ can be read as “the principal square root of *n*” or “radical *n*”
- It is the positive number which can be multiplied by itself to result in a product of “n”.



- Complete page227 #3a

Simplifying Square Roots

Simplifying Square Roots

STEPS	EXAMPLE 1 $\sqrt{8}$	EXAMPLE 2 $\sqrt{63}$	EXAMPLE 3 $\sqrt{162}$
A) Rewrite the radicand as a product of the greatest perfect square and its coinciding factor	A) $\sqrt{8} = \sqrt{4 \bullet 2}$	A) $\sqrt{63} = \sqrt{9 \bullet 7}$	A) $\sqrt{162} = \sqrt{81 \bullet 2}$
B) Split the two factors apart	B) $\sqrt{4 \bullet 2} = \sqrt{4} \bullet \sqrt{2}$	B) $\sqrt{9 \bullet 7} = \sqrt{9} \bullet \sqrt{7}$	B) $\sqrt{81 \bullet 2} = \sqrt{81} \bullet \sqrt{2}$
C) Simplify the perfect square under the square root.	C) $\sqrt{4} \bullet \sqrt{2} = 2\sqrt{2}$	C) $\sqrt{9} \bullet \sqrt{7} = 3\sqrt{7}$	C) $\sqrt{81} \bullet \sqrt{2} = 9\sqrt{2}$

Simplify the following square roots.

1) $\sqrt{27}$

2) $\sqrt{32}$

3) $\sqrt{125}$

4) $\sqrt{128}$

5) $\sqrt{600}$

6) $\sqrt{80}$

7) $\sqrt{108}$

8) $\sqrt{75}$

ANSWERS

Simplify the following square roots.

1) $\sqrt{27}$

$3\sqrt{3}$

$(\sqrt{9} \cdot \sqrt{3})$

2) $\sqrt{32}$

$4\sqrt{2}$

$(\sqrt{16} \cdot \sqrt{2})$

3) $\sqrt{125}$

$5\sqrt{5}$

$(\sqrt{25} \cdot \sqrt{5})$

4) $\sqrt{128}$

$8\sqrt{2}$

$(\sqrt{64} \cdot \sqrt{2})$

5) $\sqrt{600}$

$10\sqrt{6}$

$(\sqrt{100} \cdot \sqrt{6})$

6) $\sqrt{80}$

$4\sqrt{5}$

$(\sqrt{16} \cdot \sqrt{5})$

7) $\sqrt{108}$

$6\sqrt{3}$

$(\sqrt{36} \cdot \sqrt{3})$

8) $\sqrt{75}$

$5\sqrt{3}$

$(\sqrt{25} \cdot \sqrt{3})$

If you have trouble simplifying radicals (this will happen if you don't know your multiplication table really well) then simplify using the following steps:

1. Break the number up into a factor tree (all prime factors)

2. Put all the prime factors under the radical sign.

3. For every pair of factors that are the same, take one out and cancel the other.

4. Multiply all the numbers that you took out and put that number before the radical sign.

5. Multiply all the numbers that you didn't take out or cancel and place that number under the radical sign.

Example using the method on the previous page:

$\sqrt{450}$

Step#1

450

$50 \cdot 9$

$25 \cdot 2 \quad 3 \cdot 3$

$5 \cdot 5$

Step#2 & Step#3

$\sqrt{5 \cdot \cancel{5} \cdot 2 \cdot 3 \cdot \cancel{3}}$

Step#4 & Step#5

$5 \cdot 3\sqrt{2}$

$= 15\sqrt{2}$

ANSWER $\rightarrow \sqrt{450} = 15\sqrt{2}$

Classwork/Homework

On the worksheet handed out, complete Exercise #1 (questions 1 and 2)

Don't forget:

→ Evaluate means to give the <u>final</u> answer.

→ Simplify means to break it down.

→ Also, to evaluate the square roots of fractions:

- square root the numerator
- square root the denominator

Ex: $\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$ (final answer)

Adding and Subtracting Square Roots

Adding & Subtracting Square Roots

TEPS	XAMPLE 1 $\sqrt{8}+\sqrt{50}-4\sqrt{5}$	XAMPLE 2 $\sqrt{27}-\frac{2}{3}\sqrt{3}$
A) Simplify each term where possible	A) $2\sqrt{2}+5\sqrt{2}-4\sqrt{5}$	A) $3\sqrt{3}-\frac{2}{3}\sqrt{3}$
B) Treat the radical like a variable and add or subtract accordingly. Example: $2\sqrt{3}+4\sqrt{3}$ is like 2X + 4X, Therefore, $2\sqrt{3}+4\sqrt{3}=6\sqrt{3}$ (2X + 4X = 6X) • If fractions are present, determine a common denominator before adding or subtracting • You can only add radicals with like radicands!	B) $7\sqrt{2}-4\sqrt{5}$	B) $\left(\frac{3}{3}\right)\frac{3}{1}\sqrt{3}-\frac{2}{3}\sqrt{3}$ $\frac{9}{3}\sqrt{3}-\frac{2}{3}\sqrt{3}$ $\frac{7}{3}\sqrt{3}$

Simplify and perform the indicated operations.

1) $3\sqrt{5}+2\sqrt{2}+2\sqrt{5}$

2) $2\sqrt{10}+\sqrt{40}$

3) $\sqrt{20}+3\sqrt{5}+2\sqrt{6}$

4) $\sqrt{200}-\sqrt{72}+2\sqrt{48}$

5) $\frac{1}{3}\sqrt{27}+\sqrt{32}+\frac{1}{2}\sqrt{50}$

6) $\frac{1}{3}\sqrt{16}+\frac{2}{5}\sqrt{45}+\frac{1}{2}\sqrt{80}$

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Ex. #2

$3y + \underline{2x} + \underline{3x} + \underline{\dot{x}}$

1) i) $7\sqrt{2} - \sqrt{2} = 6\sqrt{2}$

m) $\underline{\sqrt{2}} + \underline{7\sqrt{2}} + \underline{3\sqrt{2}} = 11\sqrt{2}$

2) g) $\cancel{6\sqrt{3}} + \cancel{4\sqrt{3}} \underline{- 5\sqrt{2}} \underline{+ 7\sqrt{2}}$

$10\sqrt{3} + 2\sqrt{2}$

h) $\cancel{12} \underline{- \cancel{6\sqrt{11}} - \cancel{5\sqrt{11}}} \cancel{+ 4}$

1, 4, 9, 16

$16 - 11\sqrt{11}$

$-11\sqrt{11} + 16$

4) a) $\sqrt{12} + \sqrt{27}$

$\to 2\sqrt{3} + 3\sqrt{3}$

$= 5\sqrt{3}$

$\sqrt{27}$

$\sqrt{9 \cdot 3}$

$3\sqrt{3}$

$\sqrt{12} =$

$\sqrt{4 \cdot 3}$

$2\sqrt{3}$

$\sqrt{12} + \sqrt{27}$

1 4 9 $\underline{16}$ $\underline{25}$ 36 49

12

$2 \cdot 6$

$2 \cdot 3$

$\sqrt{2 \cdot \cancel{2} \cdot 3}$

$2\sqrt{3}$

27

$9 \cdot 3$

$3 \cdot 3$

$\sqrt{3 \cdot \cancel{3} \cdot 3}$

$3\sqrt{3}$

c) $\sqrt{32} + \sqrt{50}$

$4\sqrt{2} + 5\sqrt{2}$

$9\sqrt{2}$

$\sqrt{32}$

$\sqrt{16 \cdot 2}$

$4\sqrt{2}$

$\sqrt{50}$

$\sqrt{25 \cdot 2}$

$5\sqrt{2}$

e) $\sqrt{98} - \sqrt{8}$

$7\sqrt{2} - 2\sqrt{2}$

$5\sqrt{2}$

$\sqrt{98}$

$\sqrt{49 \cdot 2}$

$7\sqrt{2}$

$\sqrt{8}$

$\sqrt{4 \cdot 2}$

$2\sqrt{2}$

p) $3\sqrt{72} + 2\sqrt{75} - 3\sqrt{27}$

$18\sqrt{2} + \underline{10\sqrt{3}} \underline{- 9\sqrt{3}}$

$18\sqrt{2} + \sqrt{3}$

$3\sqrt{72}$

$\sqrt{36 \cdot 2}$

$3(6\sqrt{2})$

$3\sqrt{27}$

$\sqrt{9 \cdot 3}$

$3(3\sqrt{3})$

$2\sqrt{75}$

$\sqrt{25 \cdot 3}$

$2(5\sqrt{3})$

5\. a) $5\sqrt{x} + 2\sqrt{x} - 4\sqrt{x} = 3\sqrt{x}$

c) $5\sqrt{9x} + 3\sqrt{4x}$

$15\sqrt{x} + 6\sqrt{x}$

$21\sqrt{x}$

$5\sqrt{9x}$

$\sqrt{9 \cdot x}$

$5(3\sqrt{x})$

$3\sqrt{4x}$

$\sqrt{4 \cdot x}$

$3(2\sqrt{x})$

#5 g) $3\sqrt{x^3} + 5\sqrt{x^3}$

$8\sqrt{x^3}$

$= 8x\sqrt{x}$

$8\sqrt{x^3}$

$\sqrt{x^2 \cdot x}$

$8(x\sqrt{x})$

h) $3\sqrt{8x^3} - 2x\sqrt{2x}$

$6x\sqrt{2x} - 2x\sqrt{2x}$

$4x\sqrt{2x}$

$3\sqrt{8x^3}$

$\sqrt{\cancel{4} \cdot 2 \cdot \cancel{x^2} \cdot x}$

$3(2x\sqrt{2x})$

Please take out the first worksheet on radicals that you had to work on (it has exercise #1, #2)

Exercise #2

3. a) $\sqrt{3} + 5 + 3\sqrt{3} - 2 = 4\sqrt{3} + 3$

2. h) $\cancel{12} - 6\sqrt{11} - 5\sqrt{11} \cancel{+4} = 16 - 11\sqrt{11}$

$-11\sqrt{11} + 16$

4. a) $\sqrt{12} + \sqrt{27}$

$2\sqrt{3} + 3\sqrt{3}$

$5\sqrt{3}$

$\sqrt{12}$

$\sqrt{4} \cdot \sqrt{3}$

$2\sqrt{3}$

$\sqrt{27}$

$\sqrt{9} \cdot \sqrt{3}$

$3\sqrt{3}$

#4 h) $5\sqrt{12} - 3\sqrt{12} - 2\sqrt{3}$

$2\sqrt{12} - 2\sqrt{3}$

$4\sqrt{3} - 2\sqrt{3}$

$= 2\sqrt{3}$

$2\sqrt{12}$

$\sqrt{4} \cdot \sqrt{3}$

$2(2\sqrt{3}) = 4\sqrt{3}$

1 4 9 16 25 36 49 64 81 100

#4 g) $2\sqrt{8} + \sqrt{18} + 4\sqrt{2}$

$4\sqrt{2} + 3\sqrt{2} + 4\sqrt{2}$

$11\sqrt{2}$

$2\sqrt{8}$

$\sqrt{4} \cdot \sqrt{2}$

$2(2\sqrt{2})$

$= 4\sqrt{2}$

$\sqrt{18}$

$\sqrt{9} \cdot \sqrt{2}$

$3\sqrt{2}$

#5a) $5\sqrt{x} + 2\sqrt{x} - 4\sqrt{x}$

$= 3\sqrt{x}$

c) $5\sqrt{9x} + 3\sqrt{4x}$

$15\sqrt{x} + 6\sqrt{x}$

$21\sqrt{x}$

$5\sqrt{9x}$

$\sqrt{9} \cdot \sqrt{x}$

$5(3\sqrt{x})$

$3\sqrt{4x}$

$\sqrt{4} \cdot \sqrt{x}$

$3(2\sqrt{x})$

$= 6\sqrt{x}$

#5 h) $3\sqrt{8x^3} - 2x\sqrt{2x}$

$6x\sqrt{2x} - 2x\sqrt{2x}$

$4x\sqrt{2x}$

$3\sqrt{8x^3}$

$\sqrt{4x^2} \cdot \sqrt{2x}$

$3(2x\sqrt{2x})$

Please take out your worksheet on Radicals.

-put your name on it

-pass it in

#3 b. $10\sqrt{3} + 5$

d. $7\sqrt{5} +$

Multiplying and Dividing Square Roots

TEPS	XAMPLE 1 $(2\sqrt{8})(3\sqrt{6})$	XAMPLE 2 $\sqrt{\frac{40}{35}}$	XAMPLE 3 $\frac{\sqrt{24}}{\sqrt{2}}$
A) Simplify each term where possible	A) $[2(2\sqrt{2})](3\sqrt{6})$ $(4\sqrt{2})(3\sqrt{6})$	A) $\sqrt{\frac{8}{7}}$	A) $\frac{\sqrt{24}}{\sqrt{2}} = \sqrt{\frac{24}{2}}$
B) Multiply or divide integers with integers and radicands with radicands. • Remember: $\sqrt{\frac{a}{b}} \Leftrightarrow \frac{\sqrt{a}}{\sqrt{b}}$	B) $(4\sqrt{2})(3\sqrt{6})$ $12\sqrt{12}$	B) $\sqrt{\frac{8}{7}} = \frac{\sqrt{8}}{\sqrt{7}}$	B) $\sqrt{\frac{24}{2}} = \sqrt{12}$
C) Simplify your answer where possible. • Note: will rationalize the denominator in math 2204/05	C) $12\sqrt{12}$ $12(2\sqrt{3})$ $24\sqrt{3}$	C) $\frac{2\sqrt{2}}{\sqrt{7}}$	C) $\sqrt{12} = 2\sqrt{3}$

Simplify and perform the indicated operations.

1) $\sqrt{\frac{1}{16}}$

2) $\sqrt{\frac{49}{100}}$

3) $(\sqrt{75})(\sqrt{48})$

4) $(\sqrt{45})\left(\frac{1}{3}\sqrt{10}\right)$

5) $\frac{30}{\sqrt{20}}$

6) $(2\sqrt{6})(\sqrt{84})$

Multiplying and Dividing Square Roots

STEPS
A) Simplify each term where possible
B) Multiply or divide integers with integers and radicands with radicands. • Remember: $\sqrt{\frac{a}{b}} \Leftrightarrow \frac{\sqrt{a}}{\sqrt{b}}$
C) Simplify your answer where possible. • ~~Note: will rationalize the denominator in math 2204/05~~

EXAMPLE 1
$(2\sqrt{8})(3\sqrt{6})$
A) $[2(2\sqrt{2})](3\sqrt{6})$ $(4\sqrt{2})(3\sqrt{6})$
B) $(4\sqrt{2})(3\sqrt{6})$ $12\sqrt{12}$
C) $12\sqrt{12}$ $12(2\sqrt{3})$ $24\sqrt{3}$

$(2\sqrt{8})(3\sqrt{6})$

4 2

$(4\sqrt{2})(3\sqrt{6})$

$12\sqrt{12}$

4 3

$24\sqrt{3}$

EXAMPLE 2 $\sqrt{\frac{40}{35}}$
A) $\sqrt{\frac{8}{7}}$
B) $\sqrt{\frac{8}{7}} = \frac{\sqrt{8}}{\sqrt{7}}$
C) $\frac{2\sqrt{2}}{\sqrt{7}}$

$$\sqrt{\frac{40}{35}} = \sqrt{\frac{8}{7}}$$

$$\frac{\sqrt{8}}{\sqrt{7}} = \frac{2\sqrt{2}}{\sqrt{7}}$$

4

2

EXAMPLE 3 $\dfrac{\sqrt{24}}{\sqrt{2}}$
A) $\dfrac{\sqrt{24}}{\sqrt{2}} = \sqrt{\dfrac{24}{2}}$
B) $\sqrt{\dfrac{24}{2}} = \sqrt{12}$
C) $\sqrt{12} = 2\sqrt{3}$

$$\frac{\sqrt{24}}{\sqrt{2}}$$

$$= \sqrt{\frac{24}{2}} = \sqrt{12}$$

$$= \sqrt{12} \begin{smallmatrix} 4 \\ 3 \end{smallmatrix}$$

$$= 2\sqrt{3}$$

Simplify and perform the indicated operations.

1) $\sqrt{\frac{1}{16}} = \frac{\sqrt{1}}{\sqrt{16}} = \frac{1}{4}$

2) $\sqrt{\frac{49}{100}}$ $\frac{7}{10}$

3) $(\sqrt{75})(\sqrt{48})$

25 3 16 3

$(5\sqrt{3})(4\sqrt{3})$

$20\sqrt{9}$

$20(3) = 60$

4) $(\sqrt{45})\left(\frac{1}{3}\sqrt{10}\right)$

5) $\frac{30}{\sqrt{20}}$

4 5

$\frac{30}{2\sqrt{5}}$

$= \frac{15}{\sqrt{5}}$

6) $(2\sqrt{6})(\sqrt{84})$